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06CS/IS34

Third Semester B.E. Degree Examination, June/July 2013
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Define the symmetric difference of any two sets. Determine the sets A and B, given that $A - B = \{1, 3, 7, 11\}$, $B - A = \{2, 6, 8\}$ and $A \cap B = \{4, 9\}$. **(04 Marks)**

b. If S and T be two subsets of $\cup$, prove that $S \cup T = S \Delta T$ if and only if S and T are disjoint. **(06 Marks)**

c. If A and B are any two sets, prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$. **(04 Marks)**

d. 75 children went to an amusement park where they could ride the merry – go – round, roller coaster and Ferri's wheel. It is known that 20 of them have taken all the 3 rides and 55 of them have taken atleast 2 of the 3 rides. Each ride cost Rs. 0.50 and the total receipt of the amusement park is Rs. 70. Determine the number of children who did not try any of the three rides. **(06 Marks)**

2 a. Define a tautology and contradiction. For the propositions verify that $[(p \wedge q) \rightarrow r) \leftrightarrow [\neg(p \wedge q) \vee r]$ is a tautology using the truth table. **(06 Marks)**

b. Prove that, for any three propositions p, q, r

i) $[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$

ii) $[p \rightarrow (q \wedge r)] \Leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$. **(08 Marks)**

c. Using the method of contradiction validate the following arguments :

$$\begin{array}{l} p \rightarrow q \\ \neg r \vee s \\ \underline{p \vee r} \\ \therefore \neg q \rightarrow s \end{array}$$

(06 Marks)

3 a. Write down the following propositions in symbolic form and find its negation

i) If all triangles are right angled, then no triangle is equiangular

ii) For all integers n, if n is not divisible by 2, then n is odd. **(08 Marks)**

b. Prove that the following argument is valid,

$$\begin{array}{l} \forall x[p(x) \rightarrow q(x)] \\ \underline{\forall x[q(x) \rightarrow r(x)]} \\ \therefore \forall x[p(x) \rightarrow r(x)] \end{array}$$

Where p(x), q(x) and r(x) are open statements that are defined for a given universe. **(06 Marks)**

c. If m is an odd integer, prove m + 11 is an even integer using :

i) Direct method

ii) Indirect method

iii) Contradiction method. **(06 Marks)**

4 a. Prove by mathematical induction that $1^2 + 3^2 + 5^2 - - - + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$. **(06 Marks)**

b. If $A_1, A_2, A_3 - - - A_n$ are any sets, using mathematical induction prove

$\overline{\left\{\bigcup_{i=1}^{n} A_i\right\}} = \bigcap_{i=1}^{n} \overline{A_i}$ for $n \geq 2$. **(06 Marks)**

c. Find an explicit formula for

i) $a_n = a_{n-1} + n, \quad a_1 = 4 \quad \text{for} \quad n \geq 2$

ii) $a_n = a_{n-1} + 3, \quad a_1 = 10 \quad \text{for} \quad n \geq 2$. **(08 Marks)**

PART – B

5 a. Define Cartesian product of two sets. For any non empty sets A, B, C prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. **(07 Marks)**

b. Let f and g be functions from R to R defied by $f(x) = ax + b$ and $g(x) = 1 - x + x^2$. If $(g \circ f)(x) = 9x^2 - 9x + 3$, determine a, b. **(06 Marks)**

c. Define invertible function. If $f : A \to B$ and $g : B \to C$ are invertible functions, then, prove that $(g \circ f) : A \to C$ is invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. **(07 Marks)**

6 a. Define a poset (partially ordered set). The directed graph for a relation R on a set $A = \{a, b, c, d\}$ is shown below :

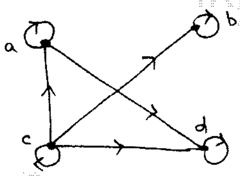

Fig. Q6(a)

i) Verify that (A, R) is a poset

ii) Draw the Hasse diagram

iii) Topologically sort the poset (A, R). **(07 Marks)**

b. Define an equivalence relation on a set. Prove that every partition of a set A induces an equivalence relation on A. **(07 Marks)**

c. Given the permutation $P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$

i) Write P as a product of disjoint cycles

ii) Compute P^{-1}, P^2 and P^3. **(06 Marks)**

7 a. Define a group. Let G be the set of all non –zero real numbers and let $a * b = \frac{ab}{2}$. Show that $(G, *)$ is an abelian group. **(07 Marks)**

b. If H and K are subgroups of a group G prove that $H \cap K$ is also a sub group of G. **(06 Marks)**

c. State and prove Lagrange's theorem. **(07 Marks)**

8 a. Define a ring. Prove that the set Z with binary operations $\oplus$ and $\otimes$ defined by $x \oplus y = x + y - 1$, $x \otimes y = x + y - xy$ is a ring. **(08 Marks)**

b. The encoding function $E : Z^2_2 \to z^5_2$ is given by the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

i) Determine all the code words

ii) Find the associated party – check matrix H

iii) Use it to decode the received words : 11101, 11011. **(07 Marks)**

c. Show that Z_5 is an integral domain. **(05 Marks)**

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